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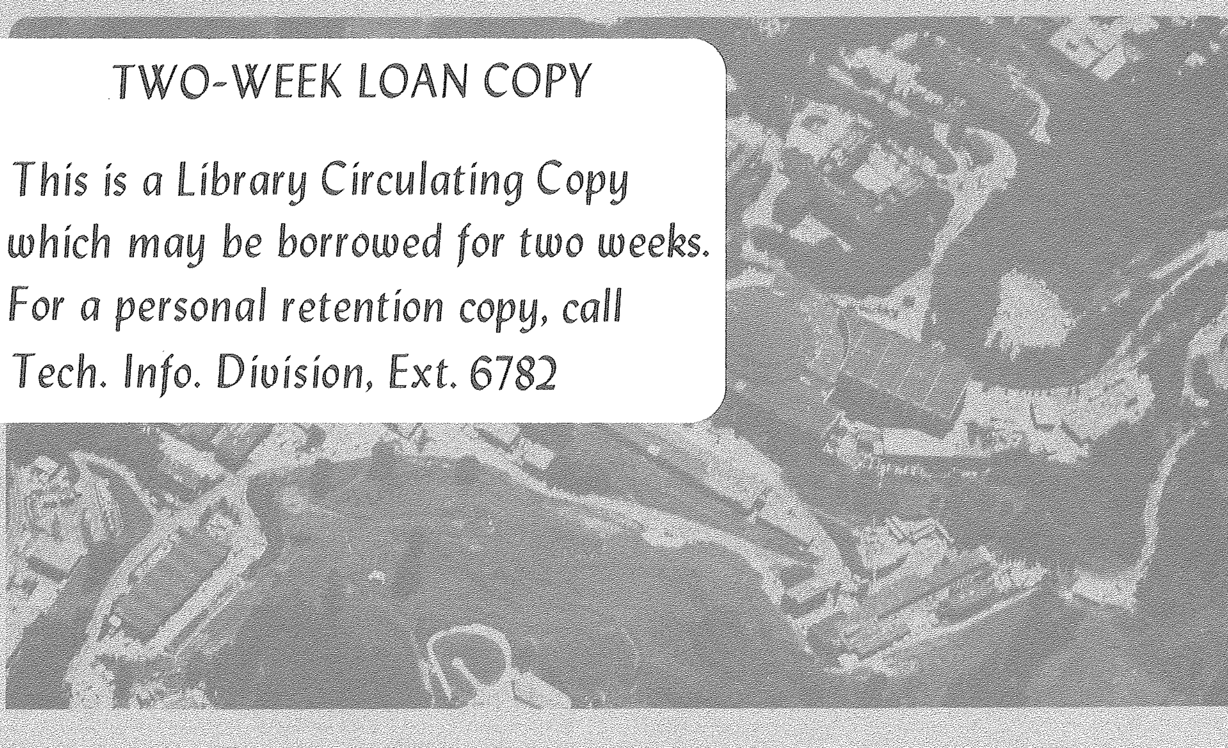

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SOFT-GLUON DYNAMICS FOR HEAVY QUARK-ANTIQUARK SYSTEMS

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Abstract

The interaction of a heavy quark-antiquark system with color gluons is classified according to the gluon energy, and a manifestly gauge-invariant multipole expansion scheme is constructed. This multipole expansion has useful applications to the study of hadronic transitions between heavy-quark-antiquark bound states, the static quark-antiquark potential and the nonperturbative structure of the gluonic vacuum.

Heavy quark systems, such as the ψ and Υ families and those of heavier quarks of possible existence, provide a useful laboratory for testing the basic structure of quantum chromodynamics. For a heavy-quark-antiquark ($Q\bar{Q}$) bound state, whose size decreases with increasing quark mass, the interaction with hard gluons can be studied perturbatively owing to asymptotic freedom when their energies are of the order of (the inverse of) the $Q\bar{Q}$ size or higher. The interaction of such a system with soft external perturbations (via gluons softer than the system size), on the other hand, is inevitably nonperturbative. In this region the difference between the system size and the interaction time may play an important role in order for an appropriate calculational framework to work, like the factorization of short- and long-distance dynamics in deep inelastic processes. Along this line, an expansion scheme, which expands the gluon fields surrounding a heavy $Q\bar{Q}$ system into multipole moments, has been developed and applied[1] to heavy-quark physics.

The purpose of this paper is to study this multipole expansion for a heavy $Q\bar{Q}$ system. A systematic classification of gluon interactions according to the gluon energy, the contruction of a manifestly gauge-invariant multipole expansion and some of its applications will be the results of the present analysis. Throughout the paper heavy $Q\bar{Q}$ mesons are assumed to be color-Coulombic bound states (for which $\alpha_S = g^2/(4\pi) < 1$) for the (strict) validity of the multipole expansion.

The one-gluon-exchange Coulomb potential is attractive $\left(-\frac{4}{3}\alpha_S/r\right)$ between a color-singlet $Q\bar{Q}$ pair while it is repulsive $\left(+\frac{1}{6}\alpha_S/r\right)$ between a color-octet $Q\bar{Q}$ pair, where r is the $Q\bar{Q}$ separation. Owing to this energy difference $\Delta\varepsilon = \frac{3}{2}\alpha_S/r$ ($\approx$ the binding energy of a color-singlet $Q\bar{Q}$ bound state), a color-octet $Q\bar{Q}$ (scattering) state is unstable and has a finite lifetime[2] $\tau \sim 1/\Delta\varepsilon \sim r/\alpha_S > r$. Single-gluon emission from (or absorption by) a color-singlet $Q\bar{Q}$ state inevitably induces a color-singlet to color-octet ($\underset{\sim}{1} \to \underset{\sim}{8}$) transition of the $Q\bar{Q}$ state, no matter how soft the emitted gluon is. This reminds us of a close analogy between the $Q\bar{Q}$ system interacting with soft gluons and a Dirac particle interacting with soft photons (as studied by means of the Foldy-Wouthuysen transformation[3]). In fact, the one-body fermion, the fermion mass and soft photons in the latter are replaced by the two-body $Q\bar{Q}$ system, the binding energy $\Delta\varepsilon$ and soft gluons in the former, respectively. This analogy is the starting point of the present investigation.

Because of the basic difference between color-singlet and color-octet $Q\bar{Q}$ states, it is natural to label soft-gluon interactions according to the associated color change of the $Q\bar{Q}$ state [i.e. $\underset{\sim}{1} \to \underset{\sim}{1}$ (color-singlet to color-singlet) or $\underset{\sim}{1} \leftrightarrow \underset{\sim}{8}$ or $\underset{\sim}{8} \leftrightarrow \underset{\sim}{8}$ transitions]. These interactions are further divided according to their ranges.

We adopt the Coulomb gauge. The simplest way to construct the color-Coulomb potential is to eliminate[4] the temporal gluon field $A_0^a(x)$ (which is a Lagrange multiplier) in the Lagrangian. Then the Coulomb potential is given by $\mathscr{D}^{ab}(\vec{x}, \vec{y}; \vec{A}) = \langle \vec{x},a|(\nabla_k[A]\nabla^k[A])^{-1}|\vec{y}, b\rangle$, where $\nabla_k^{ab}[A] = \delta^{ab}\partial_k + g\, f^{acb}A_k^c$. The emission of transverse gluons from Coulomb gluon lines is a feature specific to the color-Coulomb potential. The Coulomb gluons fall into three groups: Those exchanged between the $Q\bar{Q}$ pair, those coupled to either Q or $\bar{Q}$, and those exchanged between trasnverse gluons. The last group belongs to the pure gluon sector. Let us denote the Hamiltonians corresponding to the other two groups by $H_{Q\bar{Q}}$ and H_{A_0}, and project them onto the $Q\bar{Q}$ two-body subspace (assuming no pair creation of heavy quarks):

$$H_{Q\bar{Q}}^{a'b'|ab} = -g^2\left[(T_c)^{a'a}(T_e^*)^{b'b}\mathscr{D}^{ce}(\vec{x}, \vec{y}; \vec{A}) + \text{self-energy}\right],$$

$$H_{A_0}^{a'b'|ab} = -g\left[A_0^c(\vec{x})(T_c)^{a'a}\delta^{b'b} - A_0^c(\vec{y})\delta^{a'a}(T_c^*)^{b'b}\right], \quad (1)$$

where $T_c = \frac{1}{2}\lambda_c$, $T_c^* = \frac{1}{2}\lambda_c^*$, and (a,b)[(a',b')] denotes the color indices of the $Q^a\bar{Q}^b$ ($Q^{a'}\bar{Q}^{b'}$) system. Here $\vec{x}(\vec{y})$ is the position of the quark (antiquark). In H_{A_0} and in what follows, A_0 stands for a functional of transverse gluons A_k,

$$A_0^a(\vec{x},t) \equiv \int d^3z\, \mathscr{D}^{ab}(\vec{x}, \vec{z}; \vec{A})\left(\nabla_k[A]\partial^0A^k(\vec{z}, t)\right)^b, \quad (2)$$

which is the Coulomb-gauge temporal gluon field in the pure gluon sector. $H_{Q\bar{Q}}$ and H_{A_0} are (spatially) nonlocal functionals of Coulomb gluons and A_k. The $Q\bar{Q}$ system is surrounded by these gluons. The gluons which closely surround the $Q\bar{Q}$ system, i.e. gluons whose momenta are as hard as the $Q\bar{Q}$ size r, will predominantly build up the $Q\bar{Q}$ binding. The gluons which are distributed over the size $1/\Delta\varepsilon$ will mainly cause the color fluctuation (i.e. $\underset{\sim}{1} \leftrightarrow \underset{\sim}{8}$ transitions) of the $Q\bar{Q}$ system. On the other hand, softer gluons (i.e. symbolically,[5] $g\vec{A} < \Delta\varepsilon$) will tend to connect this fluctuating $Q\bar{Q}$ system with external perturbations.

Our next task is to construct effective soft-gluon interactions out of $H_{Q\bar{Q}}$ and H_{A_0}; this is achieved by selective summation of the contribution of hard gluons ($g\vec{A} \gtrsim \Delta\varepsilon$) in these Hamiltonians. A natural expansion scheme that emerges is a multipole expansion of the gluon field around the $Q\bar{Q}$ system, developed in powers of ρ = ($Q\bar{Q}$ size)/$(1/\Delta\varepsilon) = r\Delta\varepsilon$. This expansion classifies into multipole moments the gluons responsible for the color fluctuation of the $Q\bar{Q}$ system (i.e. $1/r > g\vec{A} \gtrsim \Delta\varepsilon$). A useful prescription for the separation of hard-gluon components is to replace, e.g., the Coulomb-gluon propagator $1/\vec{k}^2$ by $1/(\vec{k}^2 + \Lambda^2)$, where Λ is a momentum cutoff of order $\Delta\varepsilon$. The propagator $1/(\vec{k}^2 + \Lambda^2)$ has suppressed, low-frequency components ($|\vec{k}| \lesssim \Lambda$). Therefore, substitution of this propagator into $H_{Q\bar{Q}}$ and H_{A_0}, and subsequent integration over $\vec{k}$ effectively sum the contribution of hard-gluon interactions and sort out soft-gluon interactions. For example, one-transverse-gluon emission from the Coulomb gluon exchanged in $H_{Q\bar{Q}}$ is written in the form (apart from obvious color matrices and an overall factor $- ig\alpha_S/r$)

$$\int_0^1 d\zeta \, \exp\left[- r\left(\Lambda^2 + \zeta(1-\zeta)\, \vec{\partial}^2\right)^{\frac{1}{2}} \right] r^k A_k^c\left(\vec{u} + (\zeta - \tfrac{1}{2})\vec{r}\right) \quad , \tag{3}$$

where $\vec{u} = \frac{1}{2}(\vec{x} + \vec{y})$, $\vec{r} = \vec{x} - \vec{y}$ and $\vec{\partial}^2 = - (\partial^2/\partial u^k \partial u^k)$.

The effective gluon-interactions thus constructed (which are now softer than the scale $\Delta\varepsilon$) may be further expanded into multipole moments; the expansion parameter is ξ = (gluon momentum)/$\Delta\varepsilon \sim g\vec{A}/\Delta\varepsilon$. This second multipole expansion provides the classification of gluon interactions softer than $\Delta\varepsilon$. In this way, we are led to a double multipole expansion of gluon interactions developed in a double-power series in ρ and ξ.

The Foldy-Wouthuysen transformation applied to the quark and antiquark sectors serves to classify the interactions of the transverse gluons directly coupled to either Q or $\bar{Q}$. It is necessary to make a suitable transformation on $H_{Q\bar{Q}}$ simultaneously. We then separate the center-of-mass and relative coordinates of the $Q\bar{Q}$ system and arrange the interactions into a multipole series in ρ and ξ.

For a Coulombic $Q\bar{Q}$ bound state (quark mass M), the binding energy $\sim \Delta\varepsilon = \frac{3}{2}\alpha_S/r \sim M\alpha_S^2 \sim \vec{p}^2/M$, where $p^k = - i\partial/\partial r^k$ is the relative $Q\bar{Q}$ momentum. Hence $r\Delta\varepsilon \sim O(\alpha_S)$, $M \sim O(\rho^{-2}\Delta\varepsilon)$, $p \sim O(\rho\Delta\varepsilon)$. (In what follows, the multipole order $O(\rho^n\xi^m\Delta\varepsilon)$ will be written as $O(n,m)$.) We assume that the $Q\bar{Q}$ system is originally at rest so that its center-of-mass momentum $P^k = - i\partial/\partial u^k$ is a result of recoil against external perturbations. Hence we assign $P^k \sim O(g\vec{A}) \sim O(0,1)$.

The last step is to cast this double multipole expansion into a gauge-invariant form. Let us make a unitary transformation $H \to H' = e^{iW}(H - i\partial/\partial t)e^{-iW}$ with

$$W^{a'b'|ab} = - \frac{1}{2} g(r\cdot A)\cdot T_+ - \frac{1}{8} g\left(r\cdot\partial(r\cdot A)\right)\cdot T_-$$

$$- \frac{1}{48} g\{(r\cdot\partial)^2 (r\cdot A)^c + \frac{1}{2} g\, f^{ceh}(r\cdot A)^e (r\cdot\partial\, r\cdot A)^h\}$$

$$\cdot (T_+)_c + \cdots \quad , \tag{4}$$

where $(T_\pm)_c = T_c \pm T_c^*$, $T_c = (T_c)^{a'a}\delta^{b'b}$, $T_c^* = \delta^{a'a}(T_c^*)^{b'b}$, $(r\cdot A)\cdot T_+ = \{r^k A_k^c(\vec{u})\}\,(T_+)_c$, etc., and $r\cdot\partial = r^k\partial/\partial u^k$. This transformation acts on the $Q\bar{Q}$ sector only.[6] The interaction of the $Q\bar{Q}$ system with soft gluons is then described by the new transformed Hamiltonian

$$H_{AQ} = H_0 + H_E + H_H \text{ with}$$

$$H_0 = 2M + \vec{p}^2/M - \frac{4}{3}\alpha_S/r + \Delta\varepsilon\cdot P_8$$

$$H_E = - gA_0\cdot T_- - \frac{1}{2} g(r\cdot E)\cdot T_+ - \frac{1}{8} g(r\cdot\nabla\, r\cdot E)\cdot T_-$$

$$- \frac{g}{48}\left((r\cdot\nabla)^2 r\cdot E\right)\cdot T_+ + \cdots \quad ,$$

$$
\begin{aligned}
H_H = &- \frac{1}{4M^3} (\vec{p}^{\,2})^2 + \frac{1}{4M} (P^k + gA^k \cdot T_-)^2 - \frac{g}{4M} \sigma_-^k H_k \cdot T_+ \\
&- \frac{g}{4M} (\vec{L} + 2\vec{S})^k H_k \cdot T_- \\
&- \frac{g}{4M^2} [\vec{S}^k (\vec{p} \times \vec{E})^k \cdot T_+ + \frac{1}{2} \sigma_-^k (\vec{p} \times \vec{E})^k \cdot T_-] \\
&- \frac{g}{4M} [\vec{S}^k (r \cdot \nabla H_k) \cdot T_+ + \frac{1}{2} \sigma_-^k (r \cdot \nabla H_k) \cdot T_-] \\
&- \frac{g}{24M} \{ p^k, r^\ell (r \cdot \nabla F_{\ell k}) \} \cdot T_+ \\
&- \frac{g}{8M} \{P^k + gA^k \cdot T_- , r^\ell F_{\ell k} \cdot T_+\} + \cdots , \qquad (5)
\end{aligned}
$$

where $(E^k)^a = (F^{k0})^a$ and $(H^k)^a = - \frac{1}{2} \varepsilon^{k\ell m} F^a_{\ell m}$ are color-electric and -magnetic fields ($F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$), respectively, $\vec{\sigma}_- = \vec{\sigma}_Q - \vec{\sigma}_{\bar{Q}}$ and $\vec{S} = \frac{1}{2} (\vec{\sigma}_Q + \vec{\sigma}_{\bar{Q}})$ are spin matrices, $\vec{L} = \vec{r} \times \vec{p}$ is the angular momentum of relative $Q\bar{Q}$ motion, and $r \cdot \nabla = r^k \nabla_k [A]$. All the gluon fields in (5) are defined at common position $\vec{u}$ and time t. P_8 is equal to one between color-octet $Q\bar{Q}$ states and zero otherwise. In the above, only terms which are independent of Λ have been kept.[7] The transformed Hamiltonian is manifestly gauge-invariant[8] up to O(3,3), except for certain O(3,1) and O(3,3) terms[9] (not shown) coming from $H_{Q\bar{Q}}$. Note that T_- is non-vanishing only between color-octet $Q\bar{Q}$ states while T_+ induces ($\underline{1} \leftrightarrow \underline{8}$) as well as ($\underline{8} \leftrightarrow \underline{8}$) transitions of the $Q\bar{Q}$ system.

The color-Coulomb interaction $gA_0 \cdot T_-$ induces a $\Delta S = 0$, $\Delta L = 0$, $\underline{8} \leftrightarrow \underline{8}$ transition of multipole-order (0,1). The color-electric-dipole (color-E1) interaction $\frac{1}{2} g(r \cdot E) \cdot T_+$ induces a $\Delta S = 0$, $\Delta L = 1$, ($\underline{1} \leftrightarrow \underline{8}$)

or ($\underline{8} \leftrightarrow \underline{8}$) transition of order (1,2). The $\frac{1}{2}$ (g/M) $\sigma_-^k H_k \cdot T_+$ term represents a color-magnetic-dipole (color-M1) interaction of order (2,2) which changes the magnitude of the total spin of the $Q\bar{Q}$ system. Another color-M1 interaction $\frac{1}{4}$ (g/M)$(\vec{L} + 2\vec{S})^k H_k \cdot T_-$ [of order (2,2)] is proportional to the "intrinsic" color-magnetic moment $\frac{1}{4}$ (g/M)$(\vec{L} + 2\vec{S})$ of the color-octet $Q\bar{Q}$ state. The $1/(4M)(\vec{P} + g\vec{A} \cdot T_-)^2$ term [of order (2,2)] represents the recoil effect.

Note that there are no direct $\underline{1} \leftrightarrow \underline{1}$ interactions up to at least order ρ^3. The original Hamiltonian, however, contained some direct $\underline{1} \leftrightarrow \underline{1}$ (gauge-noninvariant) interactions of order (2,2), which have been removed by the unitary transformation. This shows how important it is to cast the miltipole expansion into a gauge-invariant form. Two color-E1 interactions along with an arbitrary number of $gA_0 \cdot T_-$ ($\underline{8} \leftrightarrow \underline{8}$) interactions in between lead to lowest-order $\underline{1} \leftrightarrow \underline{1}$ transitions of order (2,4). These are the "allowed" $\Delta S = 0$, $\Delta L = 0$ or 2, hadronic transitions in a heavy $Q\bar{Q}$ family such as $2\,^3S_1 \to 1\,^3S_1$ + gluons. A color-E1 interaction combined with a color-M1 interaction $\left(\frac{1}{4} (g/M)\sigma_- H \cdot T_+\right)$ induces parity-changing, $\Delta S = 1$ hadronic transitions of order (3,4) such as $^3P \to {}^1S$ + gluons. In order of magnitude, the relative rate $\Gamma(^3P \to {}^1S)/\Gamma(^3S_1 \to {}^3S_1) \simeq (r\Delta\varepsilon)^2 \simeq \alpha_s^2$ apart from the phase-space difference. Parity-changing, $\Delta S = 0$ hadronic transitions require at least three E1 interactions and are of order (3,6).

The Hamiltonian (5) represents effective soft-gluon interactions, but it is in a sense still at the tree level as to the treatment of hard-gluon interactions ($g\vec{A} \gtrsim 1/r$ and $1/r > g\vec{A} \gtrsim \Delta\varepsilon$). The successive inclusion of higher-order hard-gluon interactions by means of the weak-coupling expansion and selective summation will lead to the successive improvement of the double multipole expansion.

The pure (soft-) gluon sector has so far been treated as a fully interacting system. The present framework will therefore be used to study the nonperturbative structure of the gluonic vacuum.[10] In particular, the gluon operator $F^2_{\mu\nu}$ as well as other higher-dimensional ones which represent long-time gluonic color fluctuations can be read from the Hamiltonian (5). In more practical applications such as the study of hadronic transitions in a $Q\bar{Q}$ family, the gluon sector may be treated perturbatively using the standard interaction picture.

The standard weak-coupling expansion is inappropriate for the study of the static $Q\bar{Q}$ potential owing to color fluctuations of the $Q\bar{Q}$ system over a period of order $1/\Delta\varepsilon$. The multipole expansion scheme naturally gives rise to rearrangement[2] of the weak coupling expansion needed for this problem. In particular, the $\alpha_S^4 \ell n\alpha_S$ term in the static potential can be reproduced in a rather straightforward manner.

Details of the construction of the multipole expansion and of its applications will be reported elsewhere.

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2. This observation has been made by T. Appelquist, M. Dine and I. J. Muzinich, Phys. Rev. D17, 2074 (1978); see also Phys. Letters 69B, 231 (1977). The static $Q\bar{Q}$ potential has also been studied by F. Feinberg, Phys. Rev. Letters 39, 316 (1977); Phys. Rev. D17, 2659 (1978); W. Fishler, Nucl. Phys. B129, 157 (1977).

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4. An equivalent procedure is to integrate over A_0^a in the path-integral representation of the transition amplitude. With the Coulomb potential of the from $(\nabla_k \nabla^k)^{-1}$, we shall deal with the T^*-product Green's functions instead of the T-product ones. This potential can be obtained form the canonical Coulomb potential $(\partial^k \nabla_k)^{-1} (\partial^\ell \partial_\ell)(\partial^m \nabla_m)^{-1}$ by going from the T-products to the T^*-products.

5. It is the combination $g\vec{A}$ that causes a momentum variation of the quark motion, as seen from the covariant derivative $\nabla_k[A]$.

6. To be precise, this unitary transformation is made in the interaction picture where the unperturbed Hamiltonian is taken to be the sum of H_0 [Eq. (5)] and the full Hamiltonian for the pure gluon sector. Hence, $\vec{E}$ and $\vec{H}$ in Eq. (5) stand for the color fields defined in the pure (soft-) gluon sector.

7. Here hard gluons are treated as point-like particles to zeroth-order approximation. The Λ-dependent terms should be taken into account as one includes higher-order hard-gluon interactions.

8. To be precise, the $gA_0 \cdot T_-$ term is combined with a time derivative to give the covariant derivative $\partial/\partial t - igA_0 \cdot T_-$ in the Schroedinger equation.

9. The Coulomb terms have been examined up to terms involving three transverse gluons or less. Up to this level, the transformed Hamiltonian contains some gauge-noninvariant terms of order (3,1) and (3,3), which, however, have a vanishing contribution between color-singlet $Q\bar{Q}$ states.

10. This connection has been studied by Voloshin, Ref. 1.